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Nonlinear Dynamics Underlying Atmospheric Predictability

by

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August 1992

I. INTRODUCTION

During the three years of this University Research Initiative, 7 papers have been published in quality scientific journals with high standards of review and 3 have been submitted for publication based on full or partial support of AFOSR 89-0462, including the work of four Ph.D. candidates (one of whom received his Ph.D. degree within the last 2 years). The following list also includes a report (partially supported by this URI) on a software package for solving a general second order parameter involved system of elliptic partial differential equations on a two dimensional domain with Dirichlet and/or Neuman boundary conditions.

A study of Baroclinic Wave Behavior over Bottom Topography Using Complex Principal Component Analysis of Experimental Data, by R. L. Pfeffer, J. Ahlquist, R. J. Kung, Y. Chang and G. Li; *J. Atmos. Sci.*, **47**, 67–81, 1990.

Effects of Wave-Wave and Wave-Mean Flow Interactions on the Evolution of a Baroclinic Wave, by A. Barcilon and T. Nathan; *Geophys. Astrophys. Fluid Dyn.*, **56**, 59–79, 1991.

Asymmetric Ekman Dissipation, Sloping Boundaries and Linear Baroclinic Instability, by H. Weng and A. Barcilon; *Geophys. Astrophys. Fluid Dyn.*, **59**, 1–24, 1991.

Reflection of Hydrostatic Mountain Waves from Spatially Nonuniform layers, by W. Blumen and A. Barcilon; *Geophys. Astrophys. Fluid Dyn.*, **58**, 25–43, 1991.

Convection with Shear Has Its Limits, by L. N. Howard: *On Fluid Mechanics and Related Matters, The Proc. of a Symp. Honoring John Miles on his Seventieth Birthday*, Scripps Institute of Oceanography, Reference Series 91 - 24, 133–140, Sept. 1991.

EXSHALL: A Turkel-Zwas Explicit Large Time-Step FORTRAN Program for Solving the Shallow-Water Equations in Spherical Coordinates by I. M. Navon and Jian Yu, *Computers in Geoscience*, **17**, 1311–1343, 1991.

A Comparison of the Impact of Two Time-Differencing Schemes on the NASA/GLAS Climate Model, by R. L. Pfeffer, I. M. Navon and X. Zou; *Mon. Wea. Rev.*, **120**, 1381–1393, 1992.

Hele Shaw Convection with Imposed Shear, Part I, by R. Krishnamurti and H. Yang; submitted to *J. Fluid Mech.*, 1992.

Hele Shaw Convection with Imposed Shear, Part II, by H. Yang and R. Krishnamurti; submitted to *J. Fluid Mech.*, 1992.

Domain Decomposition and Parallel Processing of a Finite Element Model of the Shallow-water Equations , by I. M. Navon and Y. Cai; submitted to *Computer Methods in Applied Mechanics and Engineering*, 1992.

Implementation of PLTMG Version 6.0 on SCRI4D VAX with IRIS 4000 Graphics, by Jian Yu, FSU publication no., FSU-SCRI-90T-124, August 27, 1990.

Section II contains the abstracts of the above listed papers. Section III contains additional information of accomplishments under URI.

Hele Shaw Convection with Imposed Shear
Part I: Boundary Layer Formulation

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Abstract

There is a general interest and need to understand the interaction of convection with imposed shear flows. In most previous studies the shear extends throughout the convecting layer with the result that the convection takes the form of rolls with their axes aligned with the imposed flow. However, it is the case when the convective flow has vorticity not aligned with the imposed flow, that interesting momentum transfer is expected between convective and imposed flows. In a well-developed shear flow with the shear confined to the boundaries, non-aligned convection appears to occur in the interior in regions of negligible shear, (as with cumulus convection in the Earth's planetary boundary layer).

The present study is one in which convection is forced by the boundaries of a Hele-Shaw cell to be aligned perpendicular to an imposed shear flow. The imposed shear flow may be a Conette flow extending throughout the convecting layer, or one confined to a boundary, depending upon the geometry of the Hele- Shaw cell. In Part I we examine the case in which the imposed shear has a boundary layer structure and study its interaction with the convecting interior. In Part II we will present the results of small but finite amplitude convection with an imposed shear.

TABLE OF CONTENTS

II. ABSTRACTS OF PAPERS

III. FURTHER COMMENTS

1. Convection:

a. Turbulent convection in a horizontal layer between fixed rigid boundaries sometimes spontaneously produces a persistent large–scale shear flow superposed on the irregular pattern of rising warm and descending cold plumes or blobs. This shear flow implies that the mean heat transport across the layer is accompanied by a mean flux of horizontal momentum. It has been suggested by Prof. R. Krishnamurti, who discovered the large–scale flow in her convection experiments around 1980, that analogous phenomena may sometimes occur in the atmospheric boundary layer and that the associated momentum flux may play a significant role in the large scale dynamics. However other things beside convection are involved in atmospheric flows, and it seems likely that a somewhat more realistic analog would be convection in a layer one of whose boundaries is moving horizontally with respect to the other: thus the fluid motion is driven mechanically as well as thermally, with accompanying fluxes of heat and momentum. In an attempt to provide some theoretical background for experimental studies of this interaction, Dr. Howard tried to extend methods previously used for ordinary convection to find bounds for heat and momentum fluxes in terms of measures of the strength of the thermal and mechanical driving. This investigation showed that there is indeed a limited region in the plane of the heat and momentum fluxed (whose boundary depends in an explicitly determined manner on the temperature and velocity differences between the upper and lower plates) outside of which the heat and momentum fluxes cannot lie. These results and their derivation are presented in the paper by L. Howard listed in section I.

b. During the grant period R. Krishnamurti and H. Yang have completed a study [illegible]le Shaw convection with imposed shear. The problem was chosen so that the cellular convection is forced by the boundaries to have its rotation axes perpendicular to the walls and so perpendicular to the imposed shear flow. They have studied the boundary layer nature of the imposed shear flow and its interaction with the cellular convection of the interior flow. They have delineated the conditions for steady and for oscillatory finite amplitude convection; computed heat flux and counter-gradient momentum flux. During this period also, H. Yang successfully defended his dissertation and has taken a post-doctoral position at the University of Chicago, Department

of Geosciences.

2. Stability:

Work by Dr. L. Howard directed toward finding simple methods to determine the number of unstable modes in hydrodynamic stability problems, referred to in a particular case in the first progress report, has proceeded somewhat further in the direction of generality insofar as parallel flows of arbitrary profile are concerned, but in the case of non-parallel flows no significant results can be reported.

In the parallel case, the numerical method of integration around the cut to determine the number of unstable modes, which was described for the case of uniform shear with constant Brunt frequency and Ekman damping in the first report, has been formulated for arbitrary profiles, and can provide, with quite simple numerical work, the number of unstable modes. This is useful as a guide for directed numerical search for unstable modes, enabling one to decide when the search has found all of them.

A second aspect of this is that there is an old result relating the number of unstable modes of a parallel homogeneous flow having one of a special class of velocity profiles to the number of negative eigenvalues of a certain related Sturm–Liouville problem; but the way that result was obtained is not applicable except for that special class of profiles. In the past year Dr. Howard has found a way similarly to relate the number of unstable modes for an arbitrary profile to a kind of generalization of a Sturm–Liouville problem. This generalization seems to have some mathematical interest on its own, though a broad overall picture such as we have for ordinary Sturm–Liouville problems has hot yet been developed. These results have been described in an invited talk at a meeting of the Mathematical Association, but have not yet been prepared for publication.

3. Rotating Flow:

The quasigeostrophic potential vorticity equation is an approximation to the full rotating fluid equations that is appropriate when the Ekman and Rossby numbers are small and the depth is nearly uniform. As mentioned in the second report, Dr. Howard has been investigating the approximation that is appropriate when the depth is no longer nearly uniform. The results, which are obtained in a way that is conceptually simple though technically some what intricate, are quite

interesting. In some ways the mathematical structure is actually simpler that for the nearly uniform depth case, but it is of a less familiar sort. The general formulation now needs to be applied to some specific examples, and Dr. Howard is currently trying to work out the details for cases relevant to some of the experiments on the annulus with a "mountain" which have been done at GFDI and to which Prof. Pfeffer and Mr. Ding are attempting to apply the quasigeostrophic potential vorticity equation.

4. Adaptive Mesh Refinement Research:

A general and modular adaptive mesh-refinement software package aimed at solving general nonlinear elliptic partial differential equations on a 2-dimensional domain with general Dirichlet and Newman boundary conditions designed by Bank (1990). The adaptive procedure of mesh-refinement was implemented using a finite-element triangulation using a picewise polynomial, using an hierchical basis multigrid iteration for solving the resulting systems of linear equations. We implemented the software package PLTMG at FSU and developed a full graphics systems using the IRIS 4000 color graphics to display the results.

2. The Turkel-Zwas explicit large-time step scheme was applied to a hemispheric barotropic model with constraint restoration of integral invariants, in spherical coordinates (Navon and Yu, 1991). A full code including specialized Fourier filtering for polar regions, a 17-point Shapiro low-pass filtering combined with a Robert filtering for the leap-frog integration were applied and fully documented.

A novel constraint restoration method due to Navon (1987) was implemented as well in order to ascertain that the quadratic integral invariants of the shallow-water equations remain almost unchanged for long-term integration. The full code was tested with various initial conditions and was run along with graphic routines and found to be robust and performing.

3. Domain decomposition and parallel processing of a finite-element model of the shallow-water equations (paper submitted to *Computer Methods in Applied Mechanics and Engineering*, 1992). A non-overlapping domain decomposition technique was applied to a two-stage Numerov-Galerkin finite-element model of the shallow-water equations over a limited area domain. A Schur-complement matrix formulation was used along with a modified interface matrix approach in order to handle the coupling between the various subdomains. A preconditioned conjugate gradient-

square accelerated iterative technique was used in order to solve the resulting systems of nonsymmetric linear equations. A class of robust and efficient boundary-probe preconditioners was employed in order to accelerate the convergence on the interfaces between the various subdomains. We then parallelized various stages of the finite-element solution of the nonlinear hyperbolic pole's describing the shallow-water equations model and implemented the parallel code on the 4 processors of the CRAY-YMP/FSU supercomputer using multitasking techniques in a dedicated environment.

A modified interface matrix algorithm approach was developed based on modified incomplete LU factorization (MILU) preconditioner to accelerate the CGS iterative nonsymmetric solver. A speed-up of almost a factor of 4 was obtained – the ratio becoming close to 4 for higher-mesh resolutions. This work generalizes the domain decomposition approach to nonlinear hyperbolic equations and presents new approaches for preconditioning the interfaces of the subdomains.

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Bank, R. E., (1990), *PLTMG-A Software Package for Solving Elliptic Partial Differential Equations*, in the Frontiers in Applied Mathematics, **7**, SIAM, Philadelphia, 1990, 164 pp.

Navon, I. M., (1987), The Bayliss-Isaacson Algorithm and the Constraint Restoration Method are Equivalent: *Meteorol. Atmos. Phys.*, **37**, *143- -152.*

IMPLEMENTATION OF PLTMG VERSION 6.0

ON SCRI4D VAX WITH IRIS 4000 GRAPHICS†

By

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1. Introduction

PLTMG is a software package for solving a general second order parameter involved system of elliptic partial differential equations on a two dimensional domain with Dirichlet and/or Neuman boundary conditions. It was originally designed by Bank and his collegues(1979b, 1982b, 1985b, 1988, 1990a) as a tentative program to study the theoretical and practical aspects of the multigrid iterative method, adaptive grid refinement and error estimation procedures, and their interation. The adaptive procedure embodied in PLTMG is based on the ideas of Babuska (1986a, 1986b, 1986c), Rheinboldt, and their co-worker (1986, 1987). The resulting systems of nonlinear equations are solved by a combination of the approximate Newton iteration (See Bank and Rose 1981, 1982) and the hierarchical basis multigrid iteration (See Bank,Dupont and Yserentant, 1988, Yserentant, 1985, 1986a, 1986b, 1986c). The mesh refinement algorithms and data structures used in PLTMG originally followed the ideas of Bank, Sherman and Weiser (1983). The *a posteriori* error estimation procedure used for the adaptive mesh refinement was developed by Bank and Weiser (1986, 1985, 1981). The pseudo-arclength continuation procedure of PLTMG results from the joint work of Bank and Chan (1986). The sparse Gaussian elimination procedure and global stiff matrix storage used for the resulting linear sparse system of solutions was the joint work of Bank and Smith (1987). The deflation technique used for dealing with instability caused by singular matrix was developed by Chan (1984).

The purpose of this report is to present the implementaion of the software package PLTMG Edition 6.0 on the Florida State University (FSU) SCRI4D Vax and to present some examples of PLTMG run on the SCRI4D Vax with the IRIS 4000 color graphics system. The main algorithms embodied in the subroutine PLTMG are explained in detail along with the relevant theoretical background and major results. Data structures used in PLTMG are also illustrated.

† This research is also supported by the Supercomputer Computations Research Institute at Florida State University through contract No. DE-FC05-85ER250000 and by the Geophysical Fluid Dynamics Institute through Grant No. AFOSR 89-0462 and AFOSR 90-0009

Domain Decomposition and Parallel Processing of a Finite–Element model of the Shallow Water Equations

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Abstract: We present non-overlapping domain decomposition techniques applied to a two-stage Numerov-Galerkin finite element model of the shallow water equations over a limited-area domain. The Schur complement matrix formulation is employed and a modified interface matrix approach is proposed to handle the coupling between subdomains. The resulting non-symmetric Schur complement matrices, modified interface matrices as well as the subdomain coefficient matrices are solved using Preconditioned Conjugate Gradient Squared (PCGS) non-symmetric iterative solvers. Various stages of the finite element solution are parallelized and the code is implemented on a four processor CRAY Y-MP supercomputer applying multitasking techniques in a dedicated environment.

1. Introduction and motivation

Recently there has been an increase in research activities in the area of parallel computing due to the advent and growth of various parallel processing architectures. The domain decomposition approach achieves the highest level of parallelism in the numerical solution of partial differential equations. Specifically, the problem

OSCILLATORY FINITE AMPLITUDE HELE SHAW CONVECTION WITH IMPOSED SHEAR FLOWS
Part II: Flow and Temperature Pattern, Energetics, Generated Mean Flow and Momentum Flux

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ABSTRACT

In this study, using a perturbation method we first obtained the steady finite amplitude solutions and their stabilities in Hele Shaw convection without imposed shear flows. We found that the signs of $R^{(4)}$ and $\sigma^{(4)}$ are opposite to signs of $R^{(2)}$ and $\sigma^{(2)}$, respectively, whereas $R^{(3)}$ and $\sigma^{(3)}$ are zero. This result shows that the stabe finite amplitude convection will vanish when the Rayleigh number is larger than a critical value. The bifurcation diagram accordingly was presented, which showed that there was another branch of stable solution when the finite amplitude increases to a critical value.

Using a modified perturbation method with a two parameter expansion and a strained time coordinate (PLK method), we successfully obtained the oscillatory finite amplitude solutions in Hele Shaw convection with imposed shear flows. The two parameters are ϵ

and Pe, where ϵ is the amplitude of the stream function to leading order, and Pe is the Peclet number, which is the ratio of the thermal diffusion time to the time for advection by the imposed shear flow. The stability of the finite amplitude solution with and without shear was tested. We found that both are stable to infinitesimal disturbances. It has been found that in all shear flows R_{10}, ω_{10}, R_{01}, R_{11}, ω_{11}, R_{12}, ω_{12}, R_{21} and ω_{22} were zero, where R_{mn} and ω_{mn} are the coefficients of $\epsilon^m Pe^n$, respectively in the Rayleigh number and the frequency expansions. However, ω_{01}, R_{20}, R_{02}, ω_{21} and R_{22} were not zero. Their analytical expressions in terms of the Prandtl number and A, which is the aspect ratio, have been obtained. ω_{01} and ω_{21} were the frequencies due to the linear and nonlinear interactions between the imposed shear flow and convection. R_{02} and R_{20} are due to the linear interaction between the imposed shear flow and convection, and the nonlinear convection interaction. We found that the nonlinear interaction between the imposed shear flow and convection would slow down the pattern propagation, namely reducing the frequency. In the Part II, we will discuss the stream function, temperature pattern, generated mean flow and momentum flux, and their variations in Hele Shaw convection with imposed shear flows.

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ASYMMETRIC EKMAN DISSIPATION, SLOPING BOUNDARIES AND LINEAR BAROCLINIC INSTABILITY

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The effects of symmetric and asymmetric Ekman dissipation on baroclinic instability, phase speed and wave structure in a linear Eady-like channel model with without oppositely sloping boundaries are examined. In the absence of sloping boundaries, symmetric Ekman dissipation has a stabilizing tendency for all waves. When the Ekman layers are asymmetric, the viscous asymmetry destabilizes waves by (i) extending the unstable waveband toward both long and short waves, and (ii) increasing their growth rates, compared with viscous symmetric case. When the asymmetric dissipation is very small, the destabilization may result in a frictional instability for short waves which are stable in inviscid case. The viscous asymmetry makes waves dispersive and their structure asymmetric about mid-depth. However, the sense of the viscous asymmetry and the sign of shear do not affect the instability, but do modify the direction of phase propagation and the shape of the wave structure.

With asymmetric Ekman dissipation and sloping boundaries, frictional instability is a mixture of three mechanisms: (i) symmetric dissipation in the presence of sloping boundaries; (ii) viscous asymmetry in the absence of sloping boundaries; and (iii) the sense of viscous asymmetry in the presence of sloping boundaries, which is sensitive to the wavenumber and the sign of shear. The slope renders the phase speed more dispersive and sensitive to the sign of shear and the sense of the viscous asymmetry. For westerly shear, the slope reduces (increases) the wave amplitude and makes the wave more baroclinic (barotropic) in the lower (upper) level. For a given wavenumber, easterly shear dynamics may be deduced from westerly shear dynamics by a proper change of the sense of the viscous asymmetry.

KEYWORDS: Baroclinic waves, Ekman dissipation, sloping boundaries.

1. INTRODUCTION

Because of its linear and nonlinear analytical simplicity, the Eady (1949) model of baroclinic instability has served as a "workhorse" to test various aspects of that instability. Yet, we feel that the various possible alternatives offered by this model, such as asymmetric Ekman dissipation and sloping boundaries in particular, could stand a thorough re-examination which we offer below to lay the foundation before proceeding with nonlinear studies.

Using a two-layer β-plane model, Holopainen (1961) considered a *single* Ekman layer at the lower boundary and found that the effect of friction reduces the amplitude of the perturbation, except within two narrow wavebands adjacent to the two inviscid

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Contribution No. 303
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EFFECTS OF WAVE–WAVE AND WAVE–MEAN FLOW INTERACTIONS ON THE EVOLUTION OF A BAROCLINIC WAVE

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The weakly nonlinear evolution of a free baroclinic wave in the presence of slightly supercritical, vertically sheared zonal flow and a forced stationary wave field that consists of a single zonal scale and an arbitrary number of meridional harmonics is examined within the context of the conventional two-layer model. The presence of the (planetary-scale) stationary wave introduces zonal variations in the supercriticality and is shown to alter the growth rate and asymptotic equilibrium of the (synoptic-scale) baroclinic wave via two distinct mechanisms: The first is due to the direct interaction of the stationary wave with the shorter synoptic wave (wave–wave mechanism), and the second is due to the interaction of the synoptic wave with that portion of the mean field that is corrected by the zonally rectified stationary wave fluxes (wave-mean mechanism). These mechanisms can oppose or augment each other depending on the amplitude and spatial structure of the stationary wave field. If the stationary wave field is confined primarily to the upper (lower) layer and consists of only the gravest cross-stream mode, conditions are favorable (unfavorable) for nonzero equilibrium of the free wave.

In addition to the time dependent heat flux generated by baroclinic growth of the free wave, its interaction with a stationary wave field consisting of two or more meridional harmonics generates time dependent heat fluxes that vary with period of the free wave. However, if the stationary wave field contains several meridional harmonics of sufficiently large amplitude, the free baroclinic wave is destroyed.

KEY WORDS: Baroclinic instability, stationary waves, wave–wave and wave–mean flow interactions.

1. INTRODUCTION

A characteristic signature of geophysical fluid systems is the presence of several mutually interacting scales of motion (e.g., Colucci *et al.*, 1981; Li *et al.*, 1986; Pfeffer *et al.*, 1990). Here we focus on the finite-amplitude interactions between *two* scales of motion that characterize the large-scale circulation of the atmosphere: the stationary planetary-scales and the transient synoptic-scales.

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Contribution No. 280
Geophysical Fluid Dynamics Institute
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A Study of Baroclinic Wave Behavior over Bottom Topography Using Complex Principal Component Analysis of Experimental Data*

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(Manuscript received 27 March 1989, in final form 26 July 1989)

ABSTRACT

Complex principal component analysis is applied to data from three laboratory experiments of flow over two-wave sinusoidal bottom topography in a thermally driven, rotating annulus of fluid. The experiments are conducted at the same imposed temperature contrast (ΔT) and at three different rotation rates (Ω). In each case, the intensity of the wave activity is maximum downstream of the two topographic ridges. The analysis, however, reveals a fundamental difference in the behavior of the waves at lower rotation rates than at the highest rotation rate. At the lower Ω's, the baroclinic waves travel over the topographic ridges with diminished intensity and amplify on the other side of each ridge, with the result that the flows downstream of the two ridges are coherent. At the largest Ω, at which the Rossby number, Ro, is very small and the friction parameter, $r = E^{1/2}/\text{Ro}$ (where E is proportional to the Ekman number), is rather large, the waves downstream of each ridge are decoupled from those downstream of the other ridge, such that there is no coherence between them. It is thought that this behavior might be related to the small Rossby radius of deformation and large effective Ekman layer dissipation associated with baroclinic waves at large rotation rates.

1. Introduction

Complex principal component (CPC) analysis can be a powerful tool in revealing behavioral characteristics of otherwise complicated flows in the atmosphere, the oceans and other geophysical systems. Discussions of the methodology and applications to the atmosphere have been published by Wallace and Dickinson (1972), Wallace (1972), Pratt and Wallace (1976), Barnett (1983), Horel (1984) and others. More recently, this form of analysis has been used by Bernadet et al. (1989) to study flows over bottom topography in laboratory experiments. The latter study, which showed that the traveling waves were modulated on the scale of the topography, sparked the first author's interest in applying CPC analysis to the experiments discussed by Li et al. (1986) and Pfeffer et al. (1989). The present paper is devoted to the analysis of three of these experiments, which were performed in a thermally driven, rotating annulus of fluid with two-wave sinusoidal bottom topography at a single imposed temperature contrast (ΔT) and different rotation rates (Ω). As in the atmosphere, one observes in such experiments a topographically forced "planetary scale" wave and a train of smaller "synoptic scale" waves arising from the baroclinic instability of the flow. Accordingly, it seems worthwhile to pursue such experiments as a means of gaining insight into the contribution of topographic barriers (in distinction to other influences such as dif-

* Geophysical Fluid Dynamics Institute Contribution Number 280.

** The experimental portion of the research reported here was done while this author was a visitor at the Geophysical Fluid Dynamics Institute, Florida State University.

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Of Fluid Mechanics and Related Matters

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A Comparison of the Impact of Two Time-differencing Schemes on the NASA–GLAS Climate Model*

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(Manuscript received 27 February 1991, in final form 25 October 1991)

ABSTRACT

This paper presents evidence of the sensitivity of a general circulation model (GCM) to the time-differencing scheme employed when the physical parameterizations and space discretization are not changed. For this purpose, two time-marching schemes—the leapfrog and the Matsuno schemes—are analyzed and tested on the National Aeronautics and Space Administration–Goddard Laboratory for Atmospheric Studies (NASA–GLAS) fourth-order GCM in terms of the stability and behavior of 2-month-averaged fields. Linear analysis suggests that Rossby waves are slightly damped and slightly accelerated when the Matsuno scheme is used and that these effects are scale selective, being smallest for the longest waves. It also suggests that such waves are accelerated less and are not damped when the leapfrog scheme is used. An empirical orthogonal function analysis of the meridional component of velocity at 46°N, keeping at least 70% of the variance, reveals less shortwave activity in the numerical solution with the Matsuno scheme but does not lend support to the conclusion that the waves are accelerated less in the solution with the leapfrog scheme.

The two-dimensional Eliassen–Palm (E–P) flux divergence and the eddy-induced mean meridional circulation are found to be stronger in the simulation with the leapfrog time-differencing scheme than in the one with the Matsuno scheme, suggesting that the transient-wave activity is damped by the Matsuno scheme. On the other hand, the three-dimensional stationary-wave activity flux in the Northern Hemisphere simulated with the Matsuno scheme is more intense than that simulated with the leapfrog scheme, indicating that the stationary waves are more robust in the integration with the Matsuno scheme.

The GCM precipitation when integrated with the leapfrog scheme is much more intense over the tropical western Pacific and the northeastern Pacific and less intense over the western North Atlantic Ocean. The kinetic energy of waves with wavenumber greater than 9 simulated by the Matsuno scheme is consistently smaller than that obtained by the leapfrog scheme. These results give evidence that climate simulations are sensitive not only to physical parameterizations of subgrid-scale processes but also to the numerical methodology employed.

1. Introduction

General circulation models (GCMs) are the most elaborate of a hierarchy of mathematical models used in the study of climate. A general review of atmospheric GCMs presented by Simmons and Bengtsson (1984) emphasizes the importance of physical processes in determining the behavior of climatic systems. GCMs have been used for seasonal simulation (Shukla et al. 1981) as well as for studying climate variability on time scales of a month and upward. In particular, studies have been conducted to determine the sensitivity of atmospheric GCMs to changes in physical mechanisms such as surface albedo (Charney 1975), sea-ice limits in the Arctic (Herman and Johnson 1978), low-frequency variability (Charney and Shukla 1981), and inadequate orographic effects (Wallace et al. 1983).

Reviews of the numerical techniques used in numerical weather prediction models and GCMs have been presented by Mesinger and Arakawa (1976) and Kasahara (1979). Numerical experiments in which the earth's climate is simulated using different numerical schemes can play a valuable role in clarifying the nature

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EXSHALL: A TURKEL–ZWAS EXPLICIT LARGE TIME-STEP FORTRAN PROGRAM FOR SOLVING THE SHALLOW-WATER EQUATIONS IN SPHERICAL COORDINATES*

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Abstract—A FORTRAN computer program is presented and documented applying the Turkel–Zwas explicit large time-step scheme to a hemispheric barotropic model with constraint restoration of integral invariants of the shallow-water equations. We then proceed to detail the algorithms embodied in the code EXSHALL in this paper, particularly algorithms related to the efficiency and stability of T–Z scheme and the quadratic constraint restoration method which is based on a variational approach. In particular we provide details about the high-latitude filtering, Shapiro filtering, and Robert filtering algorithms used in the code. We explain in detail the various subroutines in the EXSHALL code with emphasis on algorithms implemented in the code and present the flowcharts of some major subroutines. Finally, we provide a visual example illustrating a 4-day run using real initial data, along with a sample printout and graphic isoline contours of the height field and velocity fields.

Key Words: Shallow-water equations, Spherical coordinates, Explicit finite-differences, Constraint restoration, Filtering techniques.

INTRODUCTION

Recently, a considerable amount of work has been dedicated and aimed at efficient integration of shallow-water equations in view of using these methods in numerical weather prediction models. In order to achieve computational accuracy and efficiency, most methods are concerned with the different time-scale of the advection and the gravity-inertia terms in the shallow-water equations model separately. Semi-implicit schemes (Robert, 1979; Burridge, 1975) and split-explicit schemes (Magazenkov, Shvets, and Shneyerov, 1971; Gadd, 1978a, 1978b) are examples of those methods. In the split-explicit schemes, a substantial computational economy is achieved when compared to usual explicit time integration schemes.

Turkel and Zwas (1979) proposed a space-splitting rather than a time-splitting algorithm for the explicit integration of the shallow-water equations. Their method is based on the fact that the fast gravity-inertia waves contain only a small fraction of the total available energy and therefore these waves can be calculated with a lower accuracy than the slow Rossby waves, that is on a coarser mesh. An application of the T–Z space split-explicit integration schemes with real initial data is presented and discussed by Navon and de Villiers (1987) and its properties are discussed further in Neta and Navon (1989). A linear transfer function analysis of the shallow-water equations in spherical coordinates for the Turkel–Zwas explicit large time-step scheme was carried out by Neta, Navon, and Yu (1990).

The purpose of this paper is to present a practical FORTRAN code, EXSHALL, which implements the T–Z explicit large time-step scheme for the shallow-water equations in spherical coordinates along with constraint restoration methods for enforcing a posteriori conservation of the integral invariants of the shallow-water equations. The computer program is explained in detail in connection with the various algorithms implemented in the code EXSHALL. This paper can be used as a user's guide to the program EXSHALL both in providing a brief description of the theory as well as detailed programming implementation.

We present here the T–Z scheme for the shallow-water equations in spherical coordinates and its related algorithmic background. Various filters used with T–Z scheme and which impact on its stability also are presented; a detailed description of the various subroutines in the code EXSHALL is presented; and a typical example of a 4-day run with the program EXSHALL is presented along with graphical output. Finally, in the Appendix the commented and documented FORTRAN source listing the code of the program EXSHALL is attached.

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Convection with Shear Has Its Limits

by

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1. Introduction

Suppose a fluid whose motion is adequately described by the Boussinesq equations occupies a horizontally infinite layer of thickness d. The relevant physical properties of the fluid are described by the thermal expansion coefficient α, the kinematic viscosity ν and the thermal diffusivity κ; in addition there is the gravitational acceleration g, directed downward. We suppose the top of the layer ($z = d$) is a rigid motionless plate held at temperature zero, while the bottom ($z = 0$) is held at temperature ΔT and is also rigid, but may be moving in a fixed direction in its plane with speed W^*. Dimensionless independent variables are introduced by using d as the unit of length, and the thermal time d^2/κ as unit of time. ΔT is chosen as unit of temperature, and κ/d as unit of velocity, as is commonly done in convection problems. With a suitably defined dimensionless pressure the dimensionless Boussinesq equations are then

$$\sigma^{-1}(\mathbf{u}_t + \mathbf{u}\cdot\nabla\mathbf{u}) + \nabla p - RT\,\mathbf{k} = \nabla^2\mathbf{u} \tag{1a}$$

$$\nabla\cdot\mathbf{u} = 0 \tag{1b}$$

$$T_t + \mathbf{u}\cdot\nabla T = \nabla^2 T \tag{1c}$$

$\mathbf{k}$ is the vertical unit vector and the parameters are the *Prandtl number* σ and the *Rayleigh number* R:

$$\sigma = \nu/\kappa \tag{2a}$$

$$R = \alpha g\Delta T d^3/(\kappa\nu) \tag{2b}$$

The boundary conditions are taken in the form

$$T = 0, \quad \mathbf{u} = 0 \qquad \text{on} \qquad z = 1 \tag{3a}$$

$$T = 1, \quad \mathbf{u} = U\,\mathbf{i} + V\,\mathbf{j} \qquad \text{on} \qquad z = 0 \tag{3b}$$

in which U and V are constants with $U^2 + V^2 = W^2 = (W^* d/\kappa)^2$; thus the *Reynolds number* based on the speed of the bottom plate is $Re = W^* d/\nu = W/\sigma$. We shall consider flows – solutions of the above equations – for which all relevant horizontal averages (denoted

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REFLECTION OF HYDROSTATIC MOUNTAIN WAVES FROM SPATIALLY NONUNIFORM LAYERS

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The steady, hydrostatic, inviscid, Boussinesq flow of a stably stratified fluid over a bell-shaped ridge is investigated within the framework of a linear model. The three layer model atmosphere introduced is such that the Brunt-Väisälä frequency is constant in each layer but the interfaces of the middle layer are allowed to vary gently in the cross-ridge direction. In essence, the model can be tuned in both *vertical* and *horizontal* directions. These cross-ridge variations can produce significant differences in both the cross-ridge surface wind and the surface drag compared to the response obtained by use of a horizontally uniform reflecting layer. These changes are sensitive to both the vertical location of the middle layer and to the slope of its lower interface at the ridge crest. Many of these features are explained by means of a conventional layered-model analysis.

KEY WORDS: Mountain waves, stratified flow, Boussinesq fluid.

1. INTRODUCTION

Typically, the static stability, expressed by the Brunt–Väisälä frequency, undergoes considerable variability in the vertical direction. Figure 1 shows that the static stability, N^2, exhibits variability over scales of 1–2 km in the troposphere. Similar features are retained between the two stations, as in the region between 6 and 8 km in panel b. In other cases, however, the apparent layering of the static stability may be localized in the vicinity of each station, and/or the same features may ascend or descend in altitude between GCT and DEN. This latter possibility, which may characterize the features in panel a, is examined in the present study.

We consider the dynamics of hydrostatic mountain waves in the presence of static stability variations in the cross-ridge direction. Such states will consist of regions bounded by x-dependent interfaces across which the Scorer parameter changes abruptly, remaining constant on either sides of these interfaces. These

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